Class IX Sub -: Mathematics

Chapter -12 Heron's Formula

Handout(Module ½)

In earlier classes, we have learnt about various plane figures such as triangles,quadrilaterals Squares,rectangles etc. We have also learnt to find the perimeters and areas of these figures

We know that Area of a triangle = ½ x b x h

For example , the sides of the right angled Δ ABC are 5 cm, 12 cm , 13 cm

$$\text{ar}(\Delta ABC) = \tfrac{1}{2} \times b \times h$$
$$= \tfrac{1}{2} \times 12 \times 5 \text{ cm}^2$$
$$= 30 \text{ cm}^2$$

*W*e could also take 5 cm as the base and 12 cm as the height.

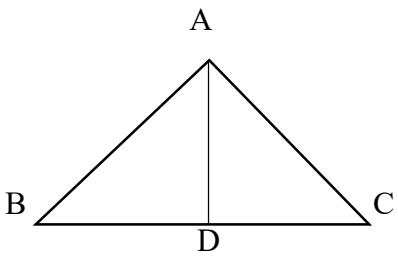
To find the area of an equilateral triangle ABC
Let AB = BC = AC = 6cm

We Need its height , if you join the midpoint of BC and vertex A we get a right angled Δ ADB and Δ ADC, and height AD :. BD = CD = 3cm

By using Pythagoras theorem. We can find the length of AD.
In Δ ADB

$$AB^2 = BD^2 + AD^2$$

$$\therefore AD^2 = AB^2 - BD^2$$
$$= 6^2 - 3^2$$
$$= 36 - 9$$



$$AD^2 = 27$$
$$AD = \sqrt{27} = 3\sqrt{3} \text{ cm}$$

Then
$$\text{ar}(\Delta ABC) = \tfrac{1}{2} \times b \times h$$
$$= \tfrac{1}{2} \times 6 \times 3\sqrt{3}$$
$$= 9\sqrt{3} \text{ cm}^2$$

To find the area of an isosceles triangle
We can calculate the area of an isosceles triangle PQR with the help of above formula, here also we need to find the height of the triangle.

eq. In Δ PQR ,

PQ=PR=8cm
and QR=6cm

Draw the perpendicular PS from P to QR,PS divides the base QR into two equal parts this is possible for equilateral triangle and isosceles triangle

In Δ PQS , by Pythagoras theorem,

$PQ^2 = QS^2 + PS^2$

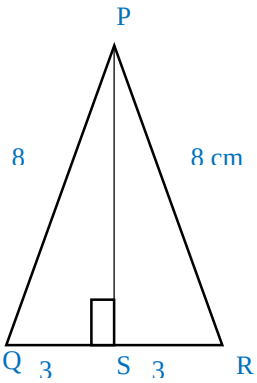
$8^2 = 3^2 + PS^2$

$PS^2 = 8^2 - 3^2$

$= 64 - 9$

$PS^2 = 55$

$PS = \sqrt{55}$ (height of Δ PQR)



∴ ar Δ ABC = ½ x b x h

$= ½ \times 6 \times \sqrt{55}\ cm^2$

$= 3\sqrt{55}\ cm^2$

Now let there be a Scalene triangle ,the length of its sides are known but the Height is not known
You will have to calculate its height and we do not have any clue to find the height
We can't use the above formula to find the area of a scalene triangle
Then, how to find the area of a triangle in terms of the lengths of its three sides?
Heron (10AD – 75AD) a greek mathematician gave a formula for finding the Area of a triangle in terms of the lengths of its three sides

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
Where s = $\frac{a+b+c}{2}$ is the semi perimeter and a , b & c are sides of the triangle

This formula is helpful where it is not possible to find the height of the triangle easily
We can use this formula to calculate the area of the triangle with three sides

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